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Report 1294

RADAR RELIEF DISPLACEMENT and RADAR PARALLAX

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ABSTRACT

This report contains the derivations of approximate formulas for radar displacement and radar parallax. The relationship between image displacement and parallax in the radar and in the photographic case is developed with the derivation of the formulas.

Photographic projection is a record of angles, whereas a radar presentation is a record of distances. Photographic relief displacement is given by the relief times the cotangent of the vertical angle; radar relief displacement is approximated by the relief times the sine of the vertical angle. Unlike photographic parallax, which is independent of target position, the direction and magnitude of radar parallax is a function of target position.

It is concluded that:

1. In most cases encountered in practice, radar relief displacements may be approximated by the formula:

$$d_R = h \sin \Phi = \frac{hH}{\sqrt{H^2 + R^2}} = \frac{hH}{S} \qquad (5)$$

2. In those cases where a more accurate solution is desired, the second approximation becomes:

$$d_R = h \sin \Phi' = \frac{h(H - h/2)}{\sqrt{(H - h/2)^2 + R^2}} = \frac{h(H - h/2)}{S_M} \qquad (4)$$

3. A usable approximation for relief computations from radar parallax is given by a single solution of the iteration formula:

$$h_{n+1} = \frac{pS_1\ S_2}{b(H - h_n/2)} \qquad (32)$$

4. The quality of the data obtained from present-day radar presentations does not warrant a quantitative investigation of these formulas, nor the design of an instrument for contouring from overlapping radar presentations.

(Note: A glossary of symbols follows.)

It is recommended that: No further development of formulas for radar relief displacement nor design of equipment for contouring from overlapping radar presentations be undertaken until the quality of data from radar presentations warrants additional investigation.

GLOSSARY OF SYMBOLS

L = Position of the aircraft

V = Position at datum vertically below aircraft (nadir point)

T = Position of the target

B = Position of the target projected to datum

LT = Distance from aircraft to target (slant range)

LB = Distance from aircraft to datum position of target

H = Height of aircraft above datum

h = Height of target above datum

R = VB = Distance from target to nadir (ground range)

d = Relief displacement

d_R = Relief displacement in the radar case

d_P = Relief displacement in the photographic case

Φ' = Vertical angle to the aircraft from a point midway between the target and the datum position

Φ = Vertical angle to the aircraft from the target

Θ = Horizontal angle at the nadir point between the target and the nadir point of the following presentation

S = LT = Distance from aircraft to target (slant range)

X = Component of ground range parallel to the air base

Y = Component of ground range perpendicular to the air base

p = Parallax

b = Distance between succeeding nadir points (air base)

S_M = Distance from aircraft to a point midway between the target and its datum position

e = Eccentricity of a conic section

h_n = nth approximation to height of target above datum

RADAR RELIEF DISPLACEMENT
and
RADAR PARALLAX

I. INTRODUCTION

1. Subject. This report contains the derivation of formulas for radar relief displacement and radar parallax, and the determination of these quantities from the formulas derived.

This investigation was conducted under authority contained in Project 8-97-90-001, "Utilization of Radar Presentations for Topographic Mapping." A copy of the project card appears as Appendix A to this report.

2. Personnel. This investigation was conducted by Randall D. Esten, under the supervision of Archer M. Wilson, Chief, Map Compilation Techniques Section, and under the general supervision of Gilbert G. Lorenz, Chief, Map Compilation Branch, and William C. Cude, Chief, Topographic Engineering Department, during the period from November 1950 to April 1951.

II. INVESTIGATION

3. Derivation of Radar Relief Displacement Formula. A radar system is fundamentally a measurer of distance. Differences in vertical height are detected by the radar system as differences in distance and recorded as a radial displacement. In Figure 1 the relief displacement is the difference in the lengths of lines LB and LT or:

$$LB - LT = d = \sqrt{H^2 + R^2} - \sqrt{(H-h)^2 + R^2} \qquad (1)$$

This equation as it stands is rather difficult to analyze, so it is expanded and simplified. Squaring and rearranging the terms, it becomes:

$$H^2 - 2hH + h^2 + R^2 = H^2 + R^2 + d^2 - 2d\sqrt{H^2 + R^2}$$

$$H^2 + R^2 = \left[\frac{d^2 + 2hH - h^2}{2d}\right]^2$$

$$H^2 + R^2 = \frac{d^4 + h^4 + 4h^2H^2 + 4d^2Hh - 2d^2h^2 - 4h^3H}{4d^2}$$

$$H^2 + R^2 = \left(\frac{4d^2Hh - 2d^2h^2 + d^4}{4d^2}\right) + \left(\frac{4h^2H^2 - 4h^3H + h^4}{4d^2}\right) \qquad (2)$$

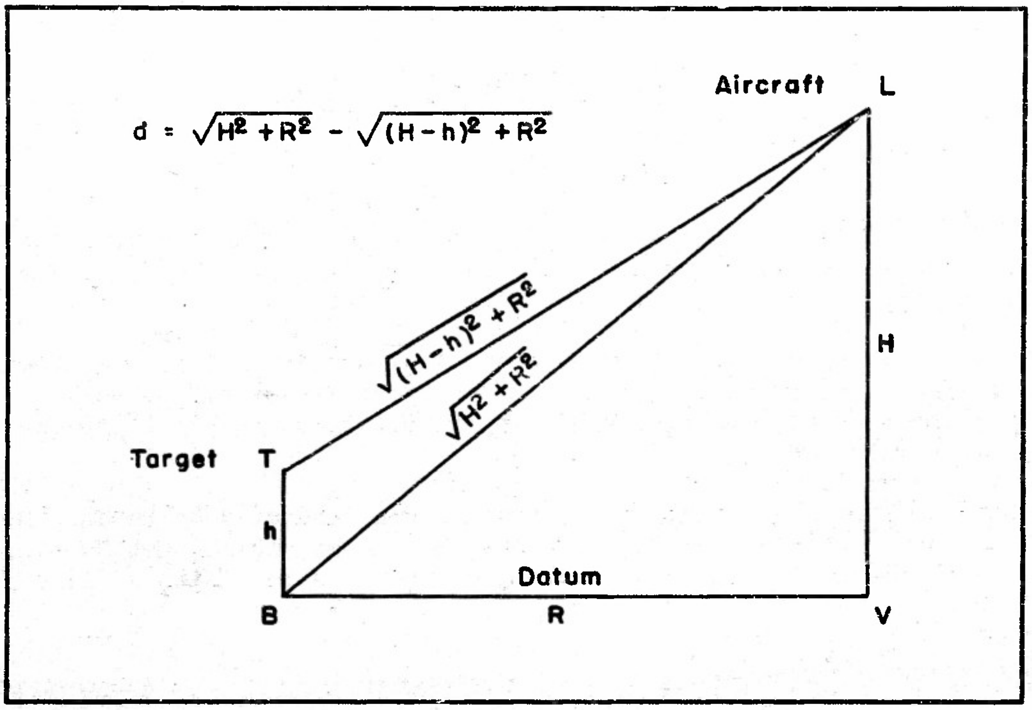


Fig. 1. Radar relief displacement.

A substitution of d^2h^2 for d^4 is to be made for the third term within the parentheses. This substitution is made to simplify the solution for d and is justified by the fact that, with the relative magnitudes involved, the numerical effects on the equation is small. Since the justification is somewhat long, the proof is carried out in Appendix B. Making this substitution, equation (2) becomes:

$$H^2 + R^2 = \frac{4d^2hH - d^2h^2}{4d^2} + \frac{4h^2H^2 - 4h^3H + h^4}{4d^2}$$

$$= Hh - \frac{h^2}{4} + \frac{h^2H^2 - h^3H + h^4/4}{d^2}$$

Rearranging the terms, taking the square root and solving for d, the expression becomes:

$$\left(H^2 - Hh + \frac{h^2}{4}\right) + R^2 = \frac{h^2}{d^2}\left(H - \frac{h}{2}\right)^2$$

$$\sqrt{\left(H - \frac{h}{2}\right)^2 + R^2} = \frac{h}{d}\left(H - \frac{h}{2}\right)$$

Since reference will be made to photographic relief displacement, distinction will be made in the symbols for the radar and photographic cases by using a subscript R for the former and subscript P for the latter. In the present case, therefore, d_R is substituted for d and becomes:

$$d_R = \frac{h\left(H - \frac{h}{2}\right)}{\sqrt{\left(H - \frac{h}{2}\right)^2 + R^2}}$$

Referring to Figure 2 it may be seen that this equation can be expressed in trigonometric form:

$$\boxed{d_R = h \sin \Phi'} \qquad \text{(second approximation)} \qquad (4)$$

where Φ' is defined as the vertical angle to the exposure station from a position midway between the elevation in question and the datum.

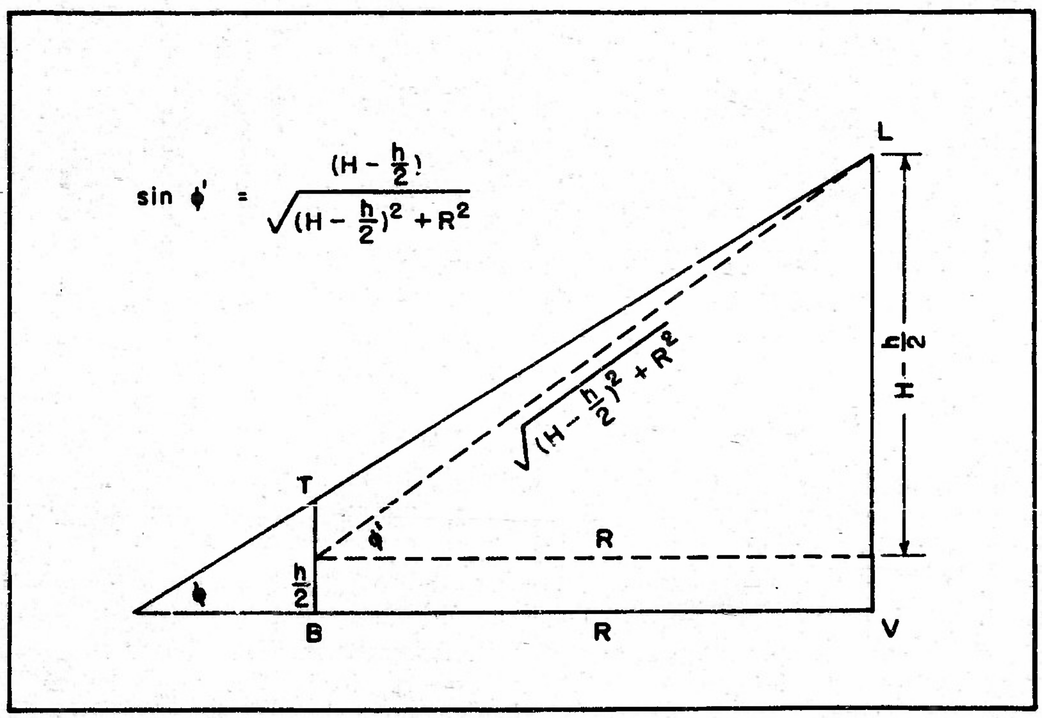


Fig. 2. Trigonometric relations (radar relief displacement).

Table I. Radar Relief Displacements and Approximation Errors

	H = 10,000 Ft											
	R = 10,000 ft			R = 20,000 ft			R = 40,000 ft			R = 80,000 ft		
Relief (h) (feet)	Relief Displacement (feet)	Error in 1st Approx. (%)	Error in 2nd Approx. (%)	Relief Displacement (feet)	Error in 1st Approx. (%)	Error in 2nd Approx. (%)	Relief Displacement (feet)	Error in 1st Approx. (%)	Error in 2nd Approx. (%)	Relief Displacement (feet)	Error in 1st Approx. (%)	Error in 2nd Approx. (%)
100	71	0.0	0.0	45	0.0	0.0	24	0.0	0.0	12	0.0	0.0
200	141	0.0	0.0	89	0.0	0.0	48	0.0	0.0	25	0.0	0.0
500	349	1.4	0.0	219	2.3	0.0	118	2.5	0.0	60	3.3	0.0
1000	689	2.6	0.0	429	4.2	0.0	231	5.2	0.0	118	5.1	0.0
2000	1336	5.8	0.1	820	9.0	0.0	439	10.5	0.0	224	10.7	0.0
4000	2480	18.2	0.6	1480	20.8	0.4	783	23.9	0.1	398	24.6	0.0
8000	3944	43.4	1.8	2261	58.2	1.7	1181	60.5	0.5	598	62.0	0.0
	H = 20,000 Ft											
	R = 20,000 ft			R = 40,000 ft			R = 80,000 ft			R = 100,000 ft		
100	71	0.0	0.0	45	0.0	0.0	24	0.0	0.0	20	0.0	0.0
200	141	0.0	0.0	89	0.0	0.0	48	0.0	0.0	39	0.0	0.0
500	351	0.9	0.0	221	1.3	0.0	120	0.8	0.0	97	1.0	0.0
1000	698	1.3	0.0	438	2.1	0.0	237	2.5	0.0	191	2.6	0.0
2000	1377	2.7	0.1	858	4.2	0.0	462	5.0	0.0	373	5.1	0.0
4000	2671	5.9	0.2	1640	9.0	0.1	878	10.5	0.0	709	10.6	0.0
8000	4960	14.1	0.7	2960	20.9	0.4	1567	23.8	0.1	1263	24.1	0.0

$$\text{Relief displacement} = \sqrt{H^2 + R^2} - \sqrt{(H - h)^2 + R^2}$$

$$\text{1st Approximation} = h \sin \Phi = \frac{hH}{\sqrt{H^2 + R^2}}$$

$$\text{2nd Approximation} = h \sin \Phi' = \frac{h\,(H - h/2)}{\sqrt{(H - \frac{h}{2})^2 + R^2}}$$

When the relief is small compared to the height, this angle may be assumed equivalent to the vertical angle to the exposure station and written as:

$$\boxed{d_R = h \sin \Phi} \qquad \text{(first approximation)} \qquad (5)$$

Table I is a comparison of the true relief displacement given by equation (1), with values from the first and second approximations.

4. Photographic Relief Displacement and Photographic Parallax. As shown in Figure 3, photographic relief displacement may be given in a form similar to equation (5); i.e.

$$d_P = h \text{ ctn } \Phi \qquad (6)$$

A comparison, or rather the contrast, of the two types of relief displacement is shown in Figure 4.

In determining relief from either a pair of photographs or radar presentations, it is not the relief displacement of a point which is measured, but the vector difference of the relief

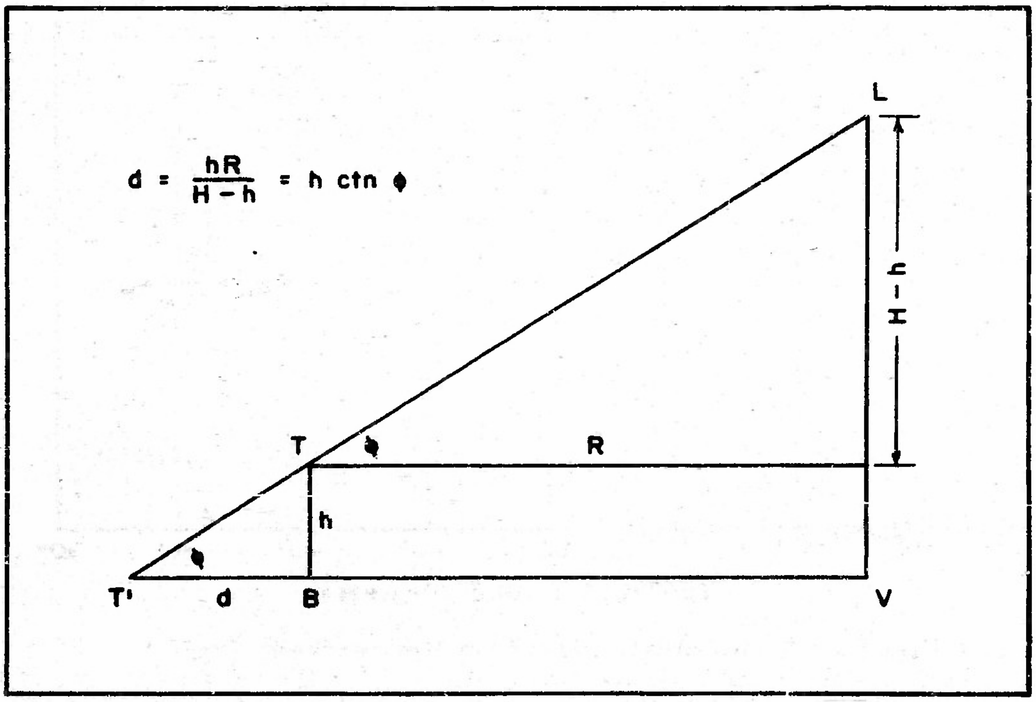


Fig. 3. Photographic-relief displacement.

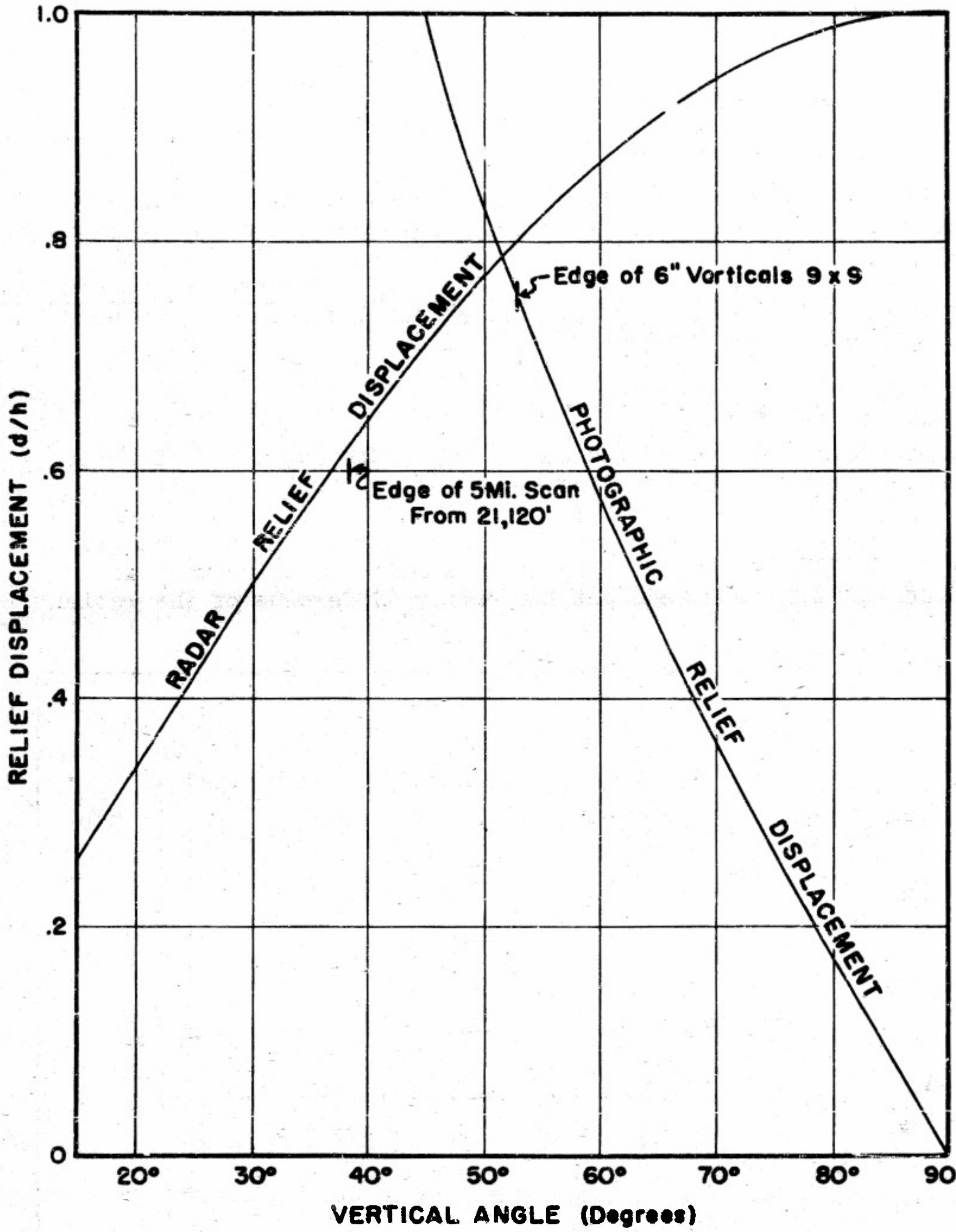


Fig. 4. Relief displacement versus vertical angle.

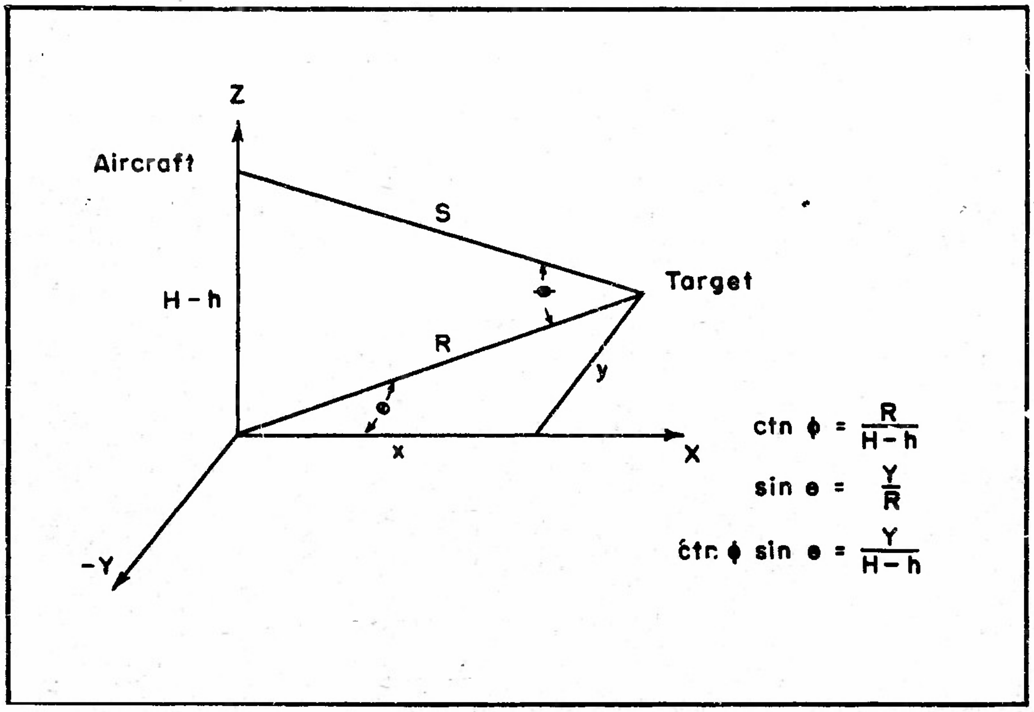


Fig. 5. Relation between vertical and horizontal angles (aerial photos).

displacement of a single point as measured on two displays made from different positions, i.e., the parallax. In describing radar parallax, it is useful and instructive to compare it to photographic parallax.

The conversion from the vertical angle (Φ) between a point and the exposure station to the horizontal angle (θ) between the air base and point position measured at the nadir, is shown in Figure 5 and given by:

$$\text{ctn } \Phi \sin \theta = \frac{Y}{H - h} \tag{7}$$

Substituting the value for ctn Φ in equation (6) for photographic relief displacement gives:

$$dp = h \text{ ctn } \Phi = \frac{h\,Y}{(H - h) \sin \theta} = \frac{hR}{(H - h)} \tag{8}$$

Taking the X and Y components of d:

$$d_{PX} = \frac{hX}{H - h} \tag{9}$$

$$d_{PY} = \frac{hY}{H - h} \tag{10}$$

Parallax is merely the difference in these displacements for a given air base. Let:

$$X_2 - X_1 = B \tag{11}$$

and since

$$Y_2 = Y_1 \tag{12}$$

therefore

$$P_x = d_{(PX)_2} - d_{(PX)_1} = \frac{(X_2 - X_1)h}{H - h} = \frac{Bh}{H - h} \tag{13}$$

$$P_y = d_{(PY)_2} - d_{(PY)_1} = \frac{(Y_2 - Y_1)h}{H - h} = 0 \tag{14}$$

In Figure 6 relief displacements are illustrated for a single target as seen from two successive exposure stations. The bottom of the target B is at the datum elevation to which the relief is referred. The top of the target T is displaced radially outward from the nadir. In each case the origin of the two plots represents the orthogonal projection of the exposure station (the nadir) V. In Figure 7 the two plots have been superimposed on the datum position of the targets. This figure represents the relief displacement for relief h at a distance Y from the air base as seen at angles θ_1 and θ_2 from the air base. Notice that the displacements are radially away from the nadir (for positive relief) and increase directly as the distance from the nadir, and that the parallax for the two exposures shown is merely the vector difference of the displacements. For vertical exposures this parallax is always parallel to the air base.

In Figure 8 another target, at the same elevation and at the same Y distance although displaced in X, is shown as seen from the same two exposure stations. In this case, although the relief displacements are dissimilar, the parallax is equal and parallel to the parallax in Figure 7. This is an illustration of formula (13) which states that the X-parallax is proportional only to the air base and the height factor h/H-h. If the Y position of the target is altered, as shown in Figure 9, the relief displacements are likewise altered, but the magnitude and direction of the parallax remains unchanged. If the relief h were increased, the displacements

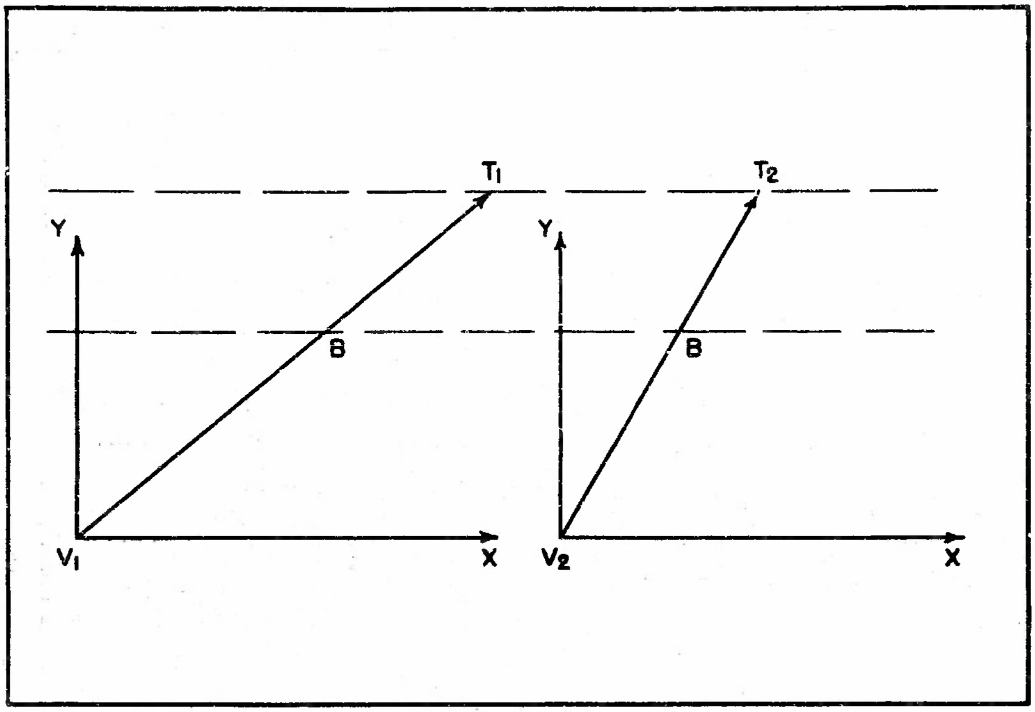


Fig. 6. Photographic relief displacement.

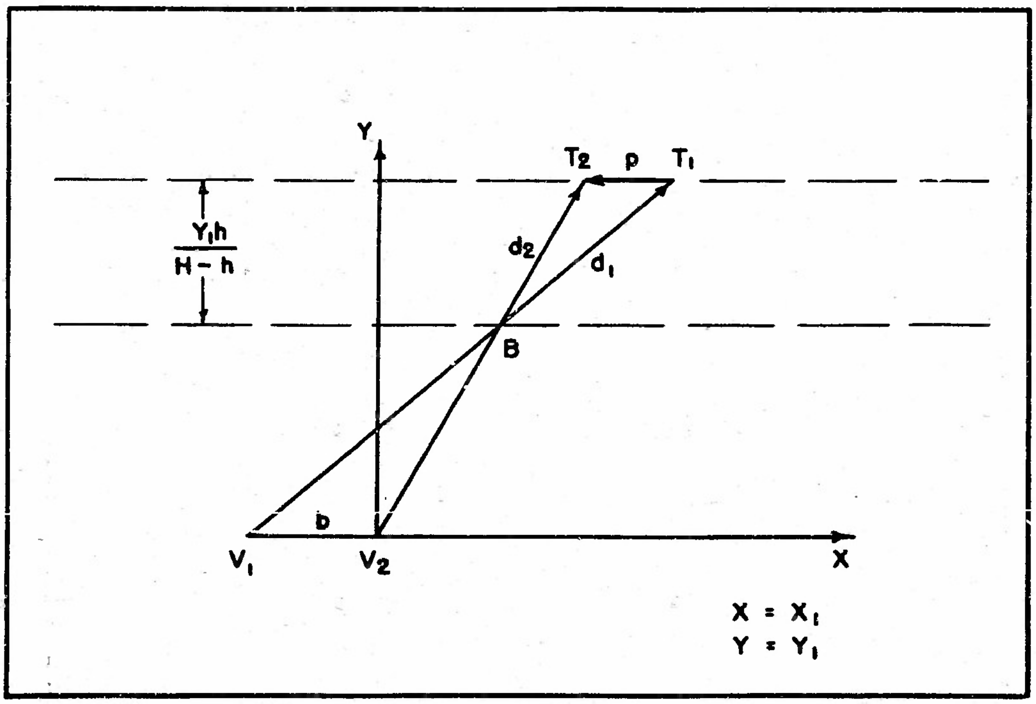


Fig. 7. Photographic parallax.

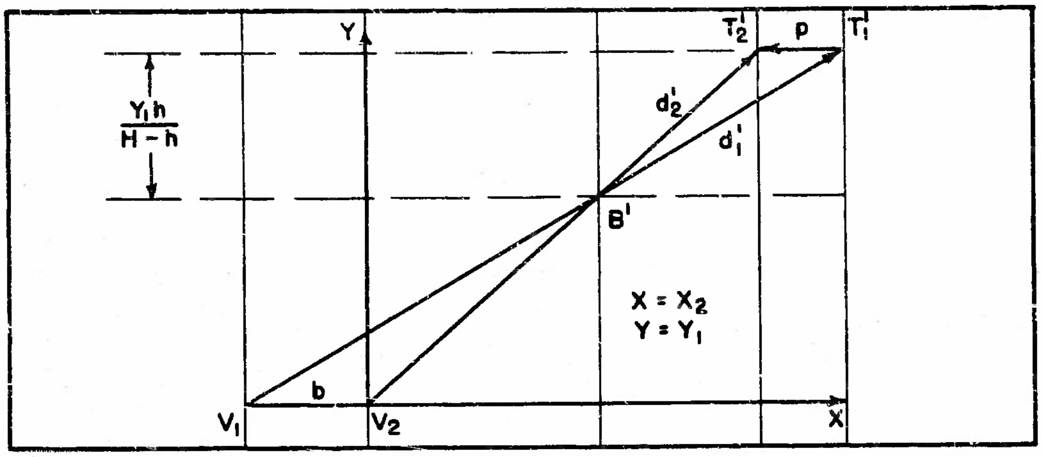


Fig. 8. Photographic parallax (Case I).

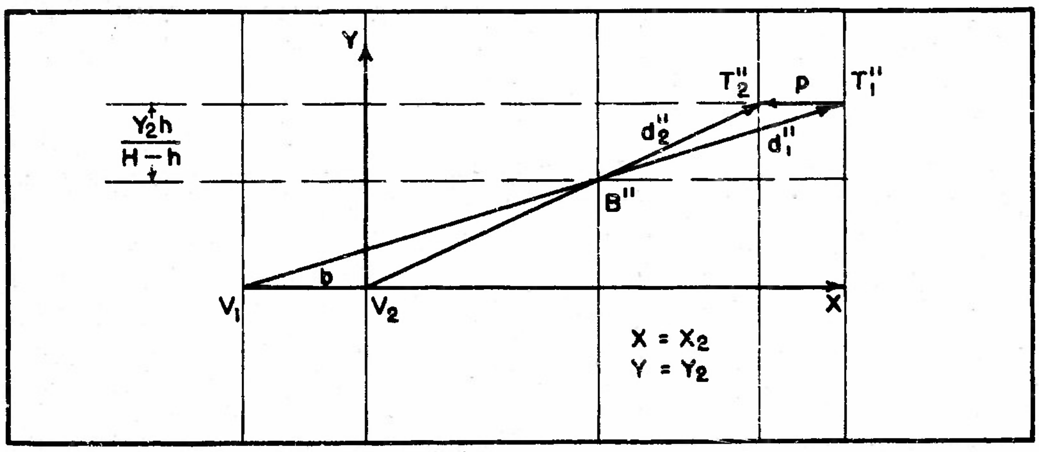


Fig. 9. Photographic parallax (Case II).

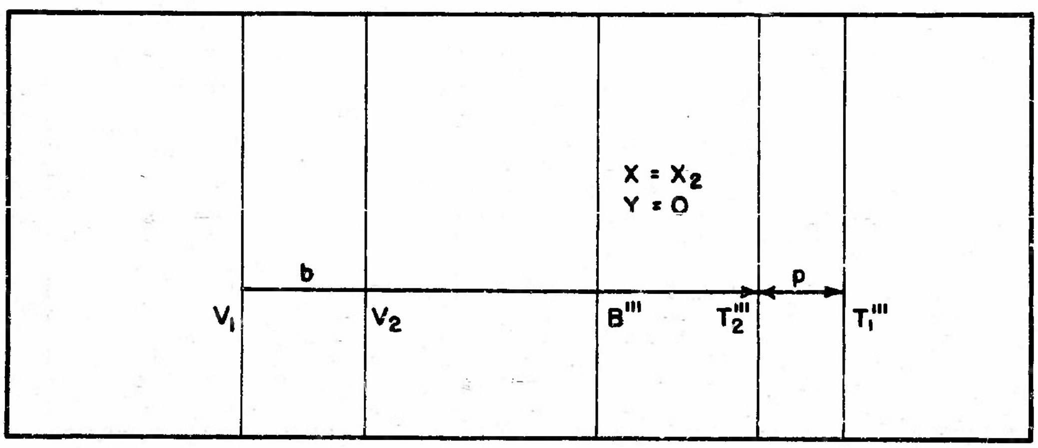


Fig. 10. Photographic parallax (Case III).

would be increased by the factor h/H-h and the parallax would be increased in the same amount and remain parallel to the air base. This is evident in Figures 8 and 9.

For the special case of Y = 0, i.e., points along the flight line, the displacement is given by the equation:

$$d_p = \frac{hX}{(H - h)} \qquad (9)$$

Figure 10 is an example of this case.

In order to illustrate the lack of dependence of the direction and magnitude of photographic parallax on target position, Figure 11 has been drawn to show relief displacements and resulting

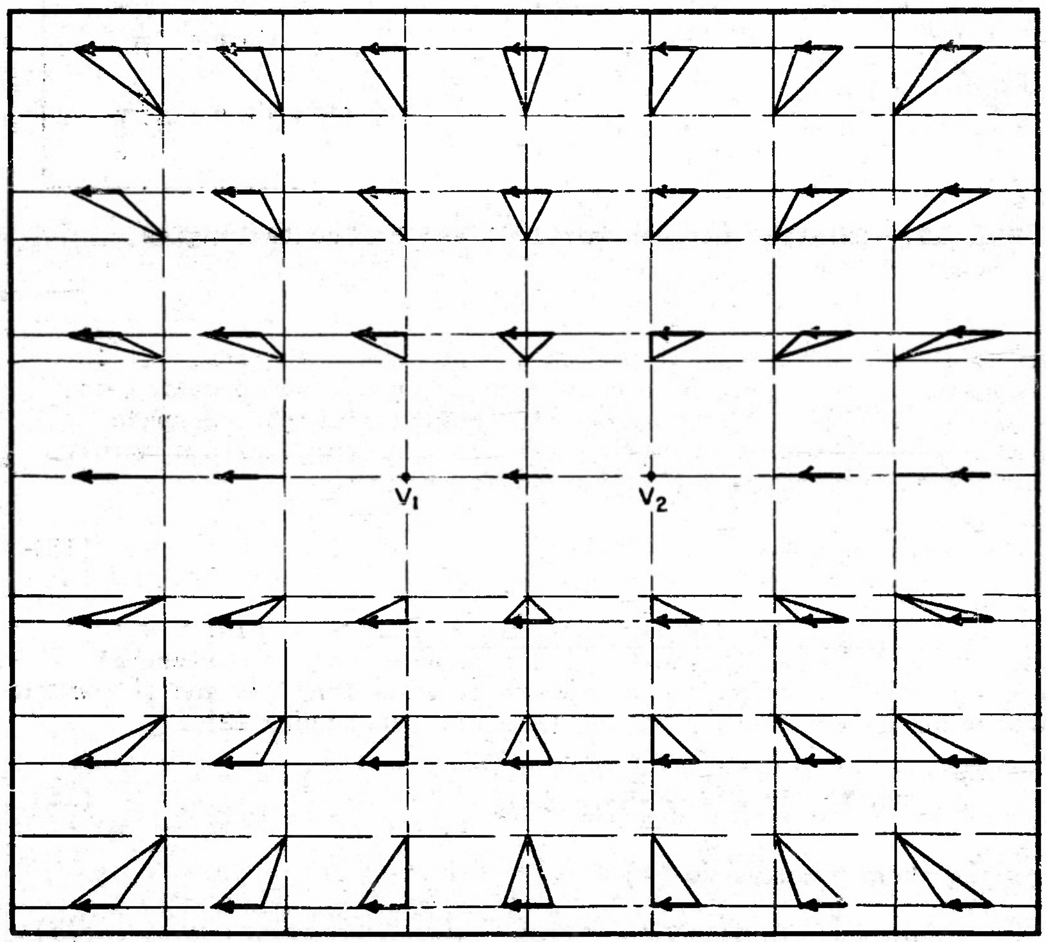


Fig. 11. Photographic parallax as a function of position.

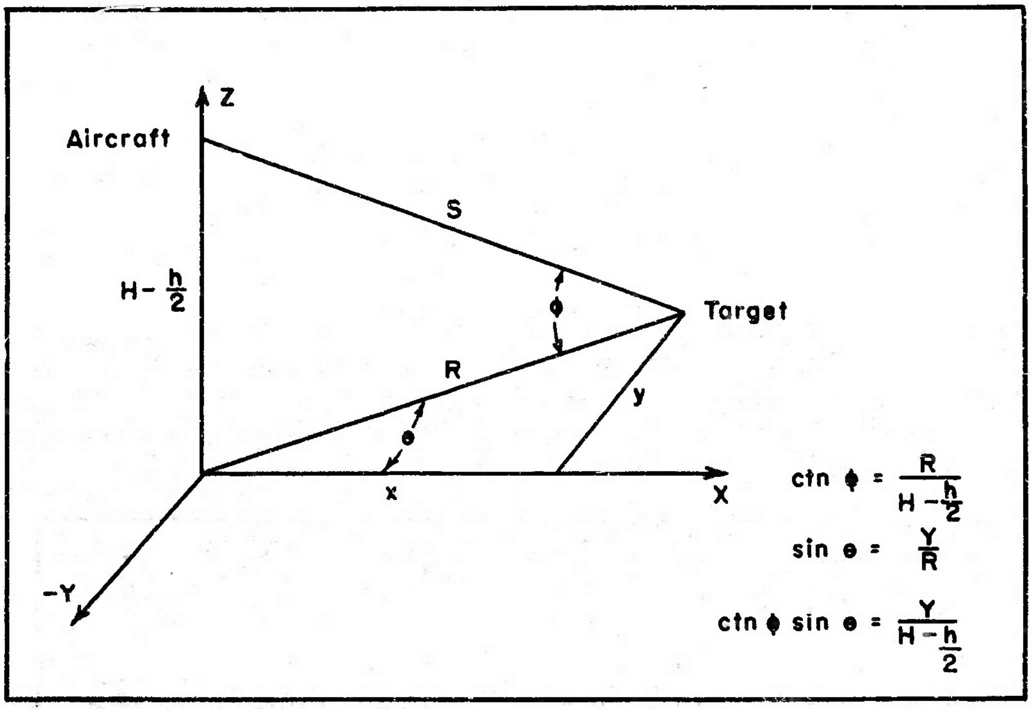


Fig. 12. Relation between vertical and horizontal angles (radar presentations).

parallax for a graticule of points at the same elevation. It is obvious that the parallax in each case is equal, and parallel to the air base V_1V_2. Thus to determine height from photographic parallax, the measured values of air base and parallax are merely substituted into formula (13) solved for h:

$$h = \frac{p\ H}{b + p} \tag{15}$$

5. Derivation of Radar Parallax Formula. Turning now to radar presentations, the conversion from vertical to horizontal angles is changed slightly to conform to equations (3) and (4) which involve an (H - h/2) in place of (H - h). In Figure 12:

$$\text{ctn}\ \Phi' \sin \Theta = \frac{Y}{H - h/2} \tag{16}$$

Applying these results to (4)

$$d_R = h \sin \Phi' \tag{4}$$

and using the identity

$$\sin \Phi' = \frac{1}{\sqrt{1 + \operatorname{ctn}^2 \Phi'}}$$

d_R becomes:

$$d_R = \frac{h}{\sqrt{1 + \left[\frac{Y}{(H - h/2)\sin\theta}\right]^2}} = \frac{h \sin\theta}{\sqrt{\sin^2\theta + \left[\frac{Y}{H - h/2}\right]^2}} \quad (17)$$

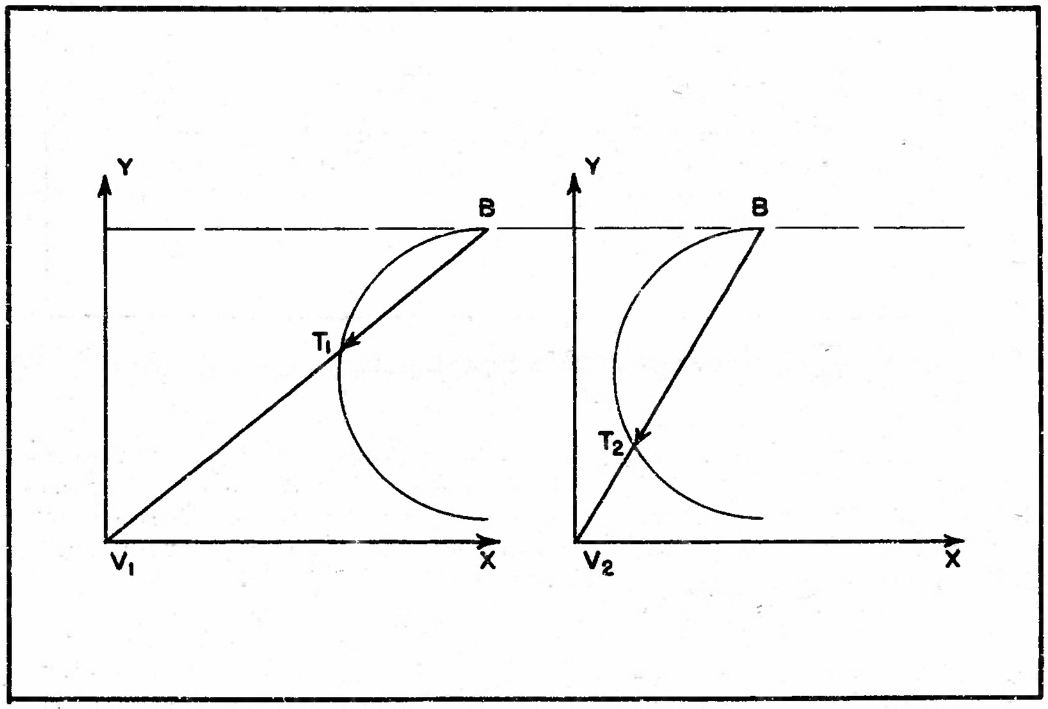


Fig. 13. Radar relief displacement.

Proceeding as in the photographic case, Figure 13 illustrates radar relief displacement for a single target as seen from two successive exposure stations. The bottom of the target, B, is again at the datum elevation to which the relief is referred. However, in the radar presentation the top of the target, T, is displaced radially inward. In each case the origin of the two plots represents the nadir points. In Figure 14 the two plots have been superimposed on the datum position of the targets. This figure represents the relief displacement for relief h at a distance Y from the air base as seen at angles θ_1 and θ_2 from the air base.

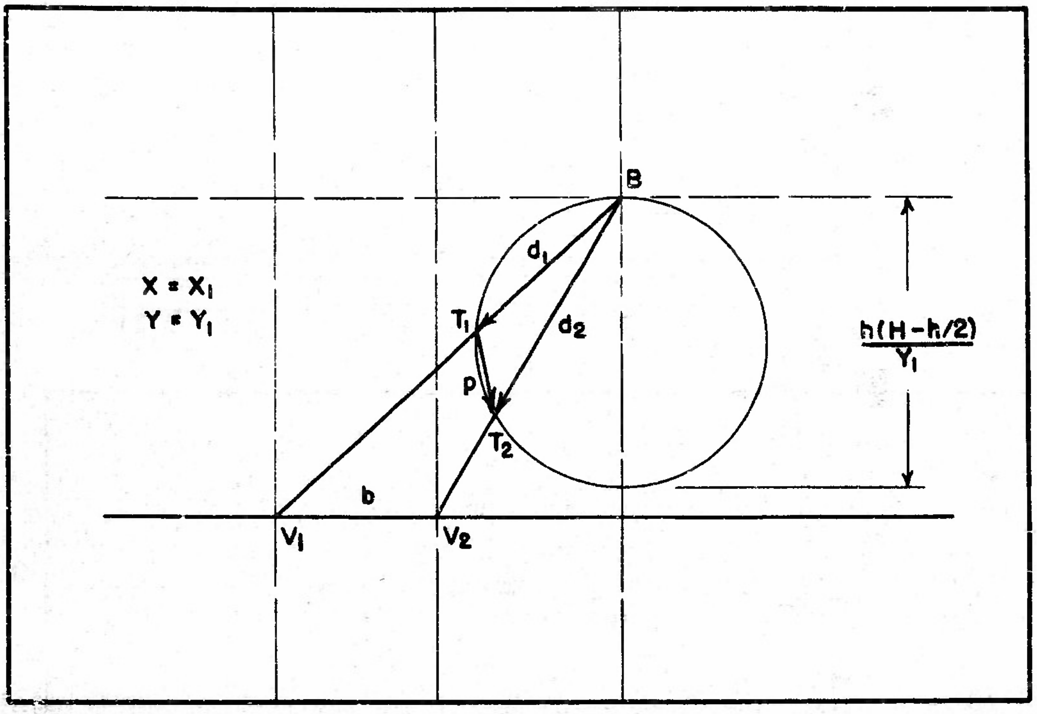


Fig. 14. Radar parallax.

Notice that the displacements are radially inward (for positive relief) and decrease directly as the distance from the nadir. Parallax for the two exposures shown is merely the vector difference of the displacements and is dependent in magnitude and direction on the position of the target in question.

In Figure 15 another target at the same elevation and at the same Y distance, but displaced in X, is shown as seen from the same two exposure stations. In this case the displacements are dissimilar and the parallax is neither equal nor parallel to the parallax of a target at the same height but in a different position. The magnitude of the parallax, however, is approximately inversely proportional to the product of the slant ranges from the two exposure stations. If the relief h were increased, the displacements would be increased by approximately the same factor, and the parallax would also be increased by this factor and would remain parallel to the parallax for a target of different elevation at the same position. This is illustrated in Figure 14. If the Y position of the target is altered as shown in Figure 16, the magnitude and direction of both the relief displacements and parallax change.

For the special case of Y = 0 (and, therefore, sin θ = 0) i.e., points along the flight line, formula (17) is indeterminate

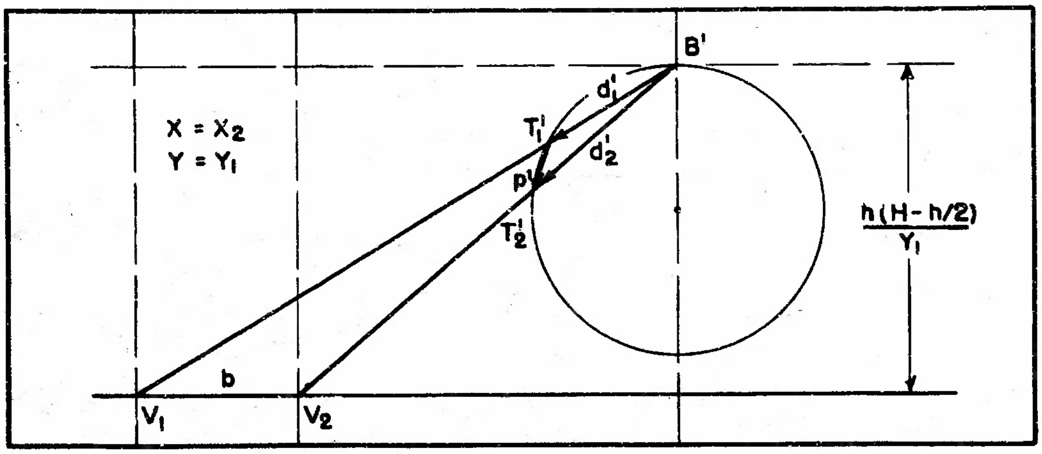


Fig. 15. Radar parallax (Case I).

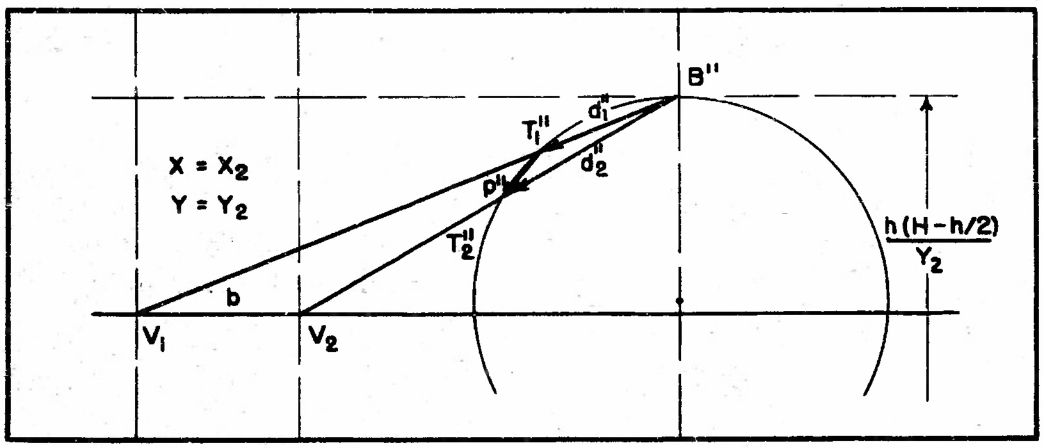


Fig. 16. Radar parallax (Case II).

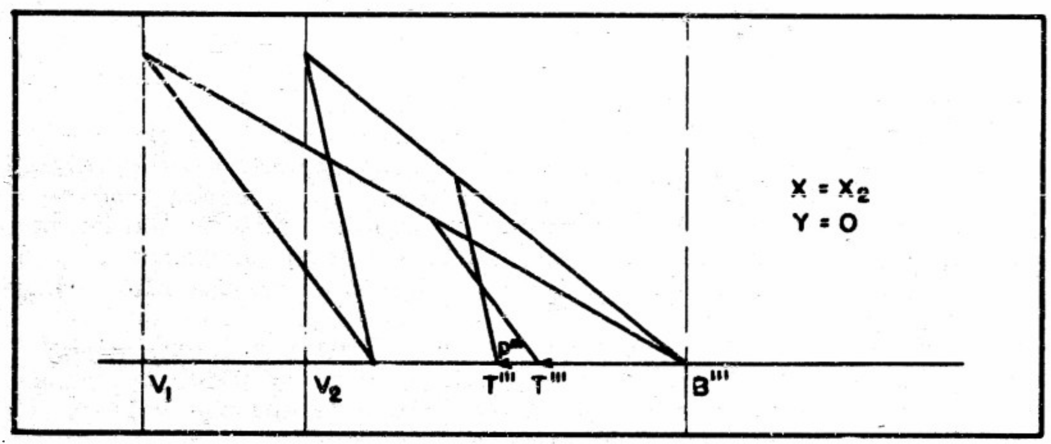


Fig. 17. Radar parallax (Case III).

and must be factored in the following manner:

$$d_R = \frac{h}{\sqrt{1 + \left[\frac{Y}{(H - h/2)\sin\Theta}\right]^2}} \tag{17}$$

$$= \frac{h\,(H - h/2)}{\sqrt{(H - h/2)^2 + R^2}}$$

$$= \frac{h\,(H - h/2)}{\sqrt{(H - h/2)^2 + X^2 + Y^2}} \tag{18}$$

$$d_R\Big|_{Y=0} = \frac{h\,(H - h/2)}{\sqrt{(H - h/2)^2 + X^2}} \tag{19}$$

Figure 17 is an example of this case.

If the slant range to the midpoint of the relief [given by $\sqrt{(H - h/2)^2 + X^2 + Y^2}$ in equation (18)] is symbolized by S_M, the relief displacement may be given by:

$$d_R \cong -\frac{h\,(H - h/2)}{S_M} \tag{20}$$

This is in distinct contrast to photographic relief displacement which is given by equation 8.

$$d_p \cong \frac{h\,R}{(H - h)} \tag{8}$$

Several important observations may be made. As might be expected, the relief displacement in both cases is proportional to the relief, but, whereas photographic relief displacement is directly proportional to the ground range and inversely proportional to the height above the point, radar displacement is directly proportional to the height above the point and inversely proportional to the slant range.

The envelope of photographic relief displacements given by equation (8) is a straight line, the distance of which from the target is a function only of the flight altitude and the relief. These lines have been shown in Figures 6-11.

The envelope of radar relief displacements as approximated by equation (20) is a circle having a diameter of h (H-h/2) /Y and passing through the target in such a way that the diameter including the target is perpendicular to the air base. The circle has been shown in Figures 13-18. However, this is only a second approximation. Referring back to equation (17) (first approximation) these curves become quadratics which fit no conic section, but resemble ellipses with an eccentricity equal to:

$$e = \frac{H - h/2}{Y + H - h/2} \qquad (21)$$

Thus, when Y is considerably greater than H, e approaches zero, and the ellipses approach circles; and when Y is zero, e is unity, and the quadratic becomes a straight line.

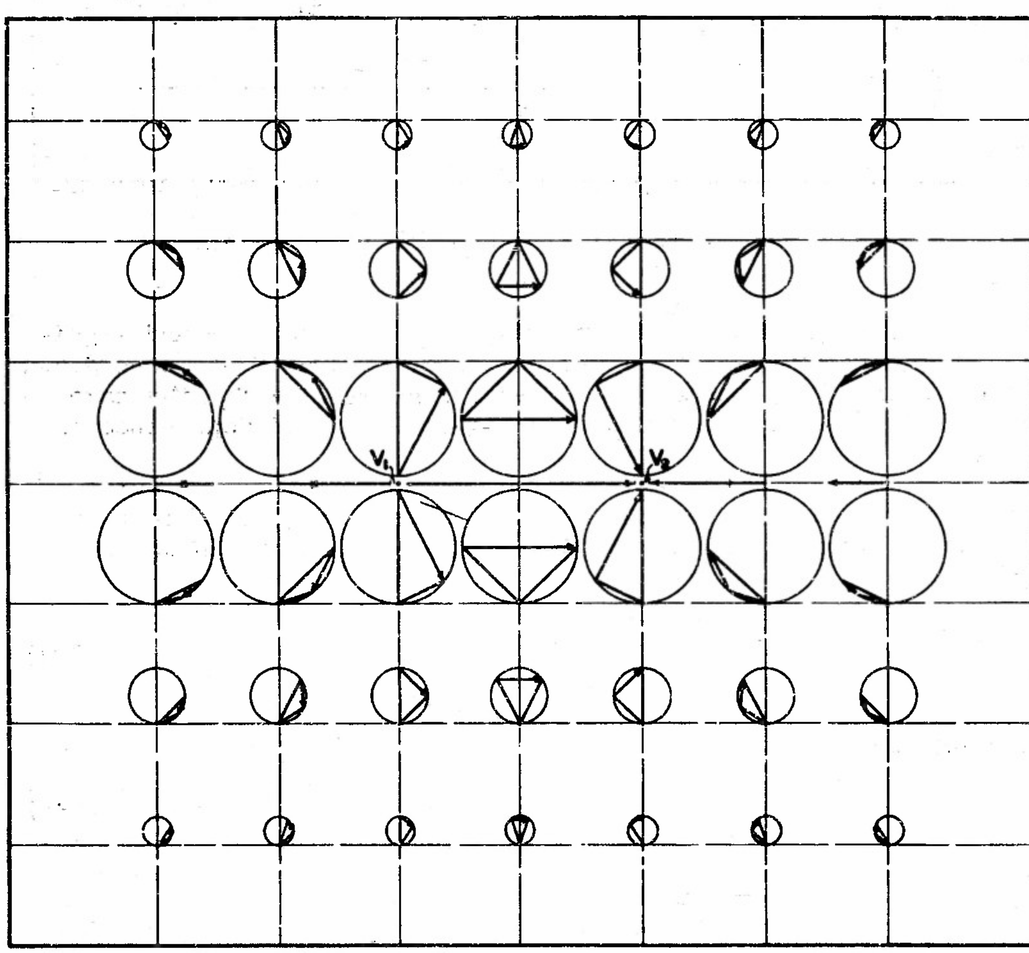


Fig. 18. Radar parallax as a function of position.

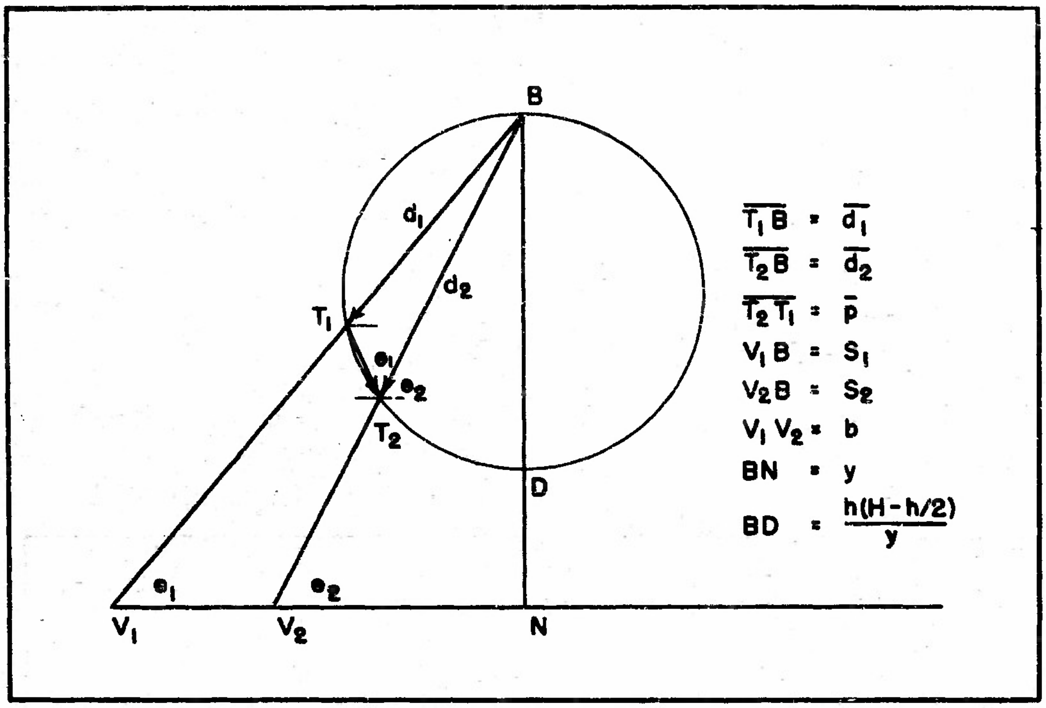


Fig. 19. Geometric relationships in radar parallax.

To illustrate the dependence of the direction and magnitude of radar parallax on target position, Figure 18 has been drawn to show relief displacements and resulting parallax for the same graticule of points illustrated in Figure 11. This dependence is clearly shown.

In order to obtain an approximate measure of height from radar parallax readings equation (20) is developed as shown in Figure 19.

$$d_1 = -\frac{h\,(H - h/2)}{S_1} \tag{22}$$

In Figure 19:

$$d_1S_1 = d_2S_2 = h(H - h/2) \tag{23}$$

therefore

$$\frac{d_1}{d_2} = \frac{S_2}{S_1} \tag{24}$$

and since angle V_1BV_2 is common to both triangles

$$V_1BV_2 \backsim T_2BT_1 \tag{25}$$

Using this fact the following statements may be made:

$$\frac{p}{b} = \frac{d_1}{S_2} \tag{26}$$

or

$$\frac{p}{b} = \frac{d_1S_1}{S_2S_1}$$

and substituting the value of d_1S_1 from equation (23), equation (26) becomes:

$$p = - \frac{h\ (H - h/2)b}{S_1S_2} \tag{27}$$

This is the magnitude of the parallax vector. The angle at which it is effective is given by:

$$\theta = \pi - (\theta_1 + \theta_2) \tag{28}$$

therefore the X-parallax is given by:

$$p_x = - \frac{h\ (H - h/2)}{S_1S_2}\ b \cos\ (\theta_1 + \theta_2) \tag{29}$$

and the Y-parallax by:

$$p_y = - \frac{h\ (H - h/2)}{S_1S_2}\ b \sin\ (\theta_1 + \theta_2) \tag{30}$$

By expanding equation (27) and solving the quadratic for the relief, it becomes:

$$\boxed{h = H - \sqrt{H^2 + \frac{2pS_1S_2}{b}}} \tag{31}$$

This equation is rather clumsy and a faster solution is obtainable by an iteration formula:

$$\boxed{h_{n+1} = \frac{p\ S_1 S_2}{b\ (H - h_n/2)}} \tag{32}$$

This formula is solved by placing $h_n = 0$, or to some approximate value if there is any elevation information available, and solving for h_{n+1}. In the second solution the value of h_{n+1} is used for h_n and the equation resolved for h_{n+1}. Generally the accuracy of the data justifies only a single solution, but the approximation converges very rapidly to the value given by equation (31) and a second solution is only a matter of a single multiplication and division. The error in the first solution will be of the order of $h/2H$.

Table II is a list of heights computed using formulas (31) and a single solution of (32) for a rectangular grid of points with a relief of 1,000 feet. Note the error (32) - (31) is very nearly constant, $h/2H = 5$ percent. In Figure 20 the solid lines illustrate lines of equal parallax for constant relief and the air base as shown. The dotted lines are lines of equal parallax direction. Note that the sets of curves are orthogonal.

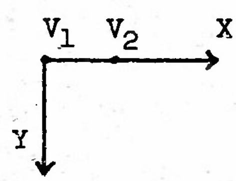
Table II. Error in Relief Computations from Parallax according to Equations (31) and (32)

H = 10,000 ft

B = 10,000 ft

h = 1,000 ft

Elevation Error in %



Y-coordinate		X-coordinate				
		10,000 (%)	20,000 (%)	30,000 (%)	40,000 (%)	50,000 (%)
0	(31)		-20.2	-9.5	-4.5	-2.8
	(32)		-23.4	-12.7	-9.1	-7.5
10,000	(31)	17.9	-5.0	-5.4	-3.6	-2.4
	(32)	11.0	-9.5	-9.9	-8.3	-7.2
20,000	(31)	7.2	1.2	-1.6	-2.0	-1.7
	(32)	1.5	-3.9	-6.4	-6.8	-6.5
30,000	(31)	3.7	1.8	-0.1	-0.7	-0.9
	(32)	-1.7	-3.4	-5.1	-5.7	-5.8
40,000	(31)	2.2	1.4	0.5	-0.1	-0.4
	(32)	-3.0	-3.7	-4.5	-5.1	-5.4
50,000	(31)	1.4	1.1	0.6	0.1	-0.1
	(32)	-3.7	-4.0	-4.5	-4.9	-5.1

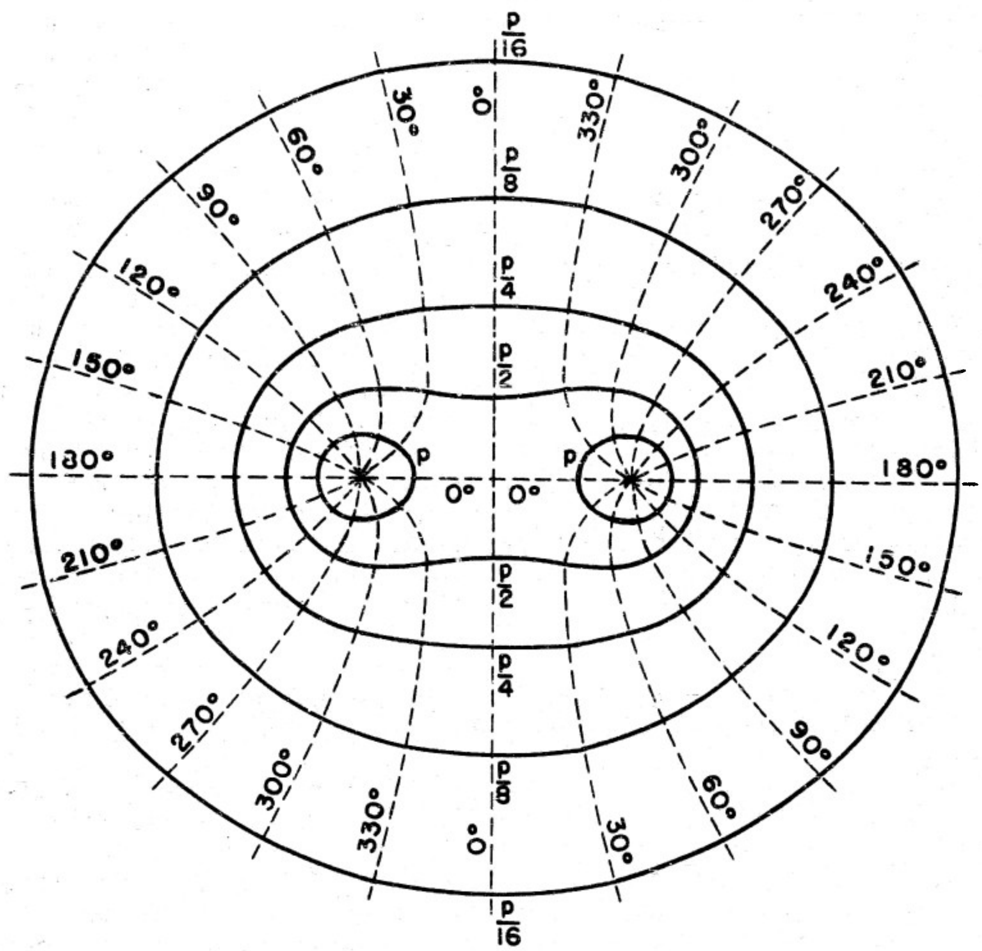


Fig. 20. Radar parallax (lines of equal magnitude and direction).

III. DISCUSSION

6. Evaluation of Formulas. From equation (32) it is apparent that for a given air base, flight height, and target position, the height is a linear function of parallax. Thus an n percent error in the measurement of parallax would result in an n percent error in the height computations. This is an important consideration in the design of any radar contouring apparatus. On 35 mm negatives the parallax, at an average range of 20,000 ft on 10 nautical-mile scans under the conditions given in Table II, is approximately 7 mils or 0.179 mm. A slight error in the measurement

of parallax would mean a considerable error in elevation. The points are not positioned accurately enough on the film in the first place to justify precise measurement. Moreover, a simple contouring instrument would in all probability measure only a component of the parallax and thus further reduce the accuracy of the results.

General consideration of image displacement for normal angular parallax reveals several unusual features. First of all, according to the preceding development, the magnitude of parallax should fall off rapidly with increasing range, while the direction of parallax should vary through nearly 360°. This indicates that if two presentations were viewed stereoscopically there would be strong height impressions 90° from the base, decreasing as the distance from the base increases. Also there would be areas of no relief impression whatsoever. However, this does not seem to be the case. The impression obtained in viewing two overlapping radar photographs in reversed position is one very similar to viewing aerial photographs in the normal position. The general relief impression, although exaggerated, corresponds to the known configuration of the ground and does not exhibit the dependences on position predicated by normal angular parallax. The reason for this seems to be due primarily to displacements caused by aspect fluctuation and the effects of radar shadows which mask the predicated effect of normal angular parallax.

Aspect fluctuation is the variation in the apparent position of a target due to the changing angle of view. In Figure 21 the radar equipped aircraft approaching from the west picks up the west wall of the target building. Interpreting from the radar information, the position of the target would be placed at position 1. Any possible returns from the remaining sides or the top would be reflected away from the aircraft. As the aircraft passes to the north, the north wall of the target registers on the radar screen and would be plotted at 2. Likewise, after passage, the east wall of the target is registered and plotted at 3. The resolution of present radars is not great enough so that the various returns can be distinguished, i.e. the return from the west wall looks just like the return from the east wall, and without further information they must be matched as identical points. If the elevation of the target and its dimensions were known these plotted positions could be corrected and the true position of the target determined. However, this is generally the type of information sought and not available beforehand.

In terms of stereoscopic vision, the brain, being unable or unwilling to distinguish between returns from the west wall and the east wall, accepts the returns as being from identical targets and fuses them, giving a false impression of position and elevation. In the general case the target will not have the rectangular shape

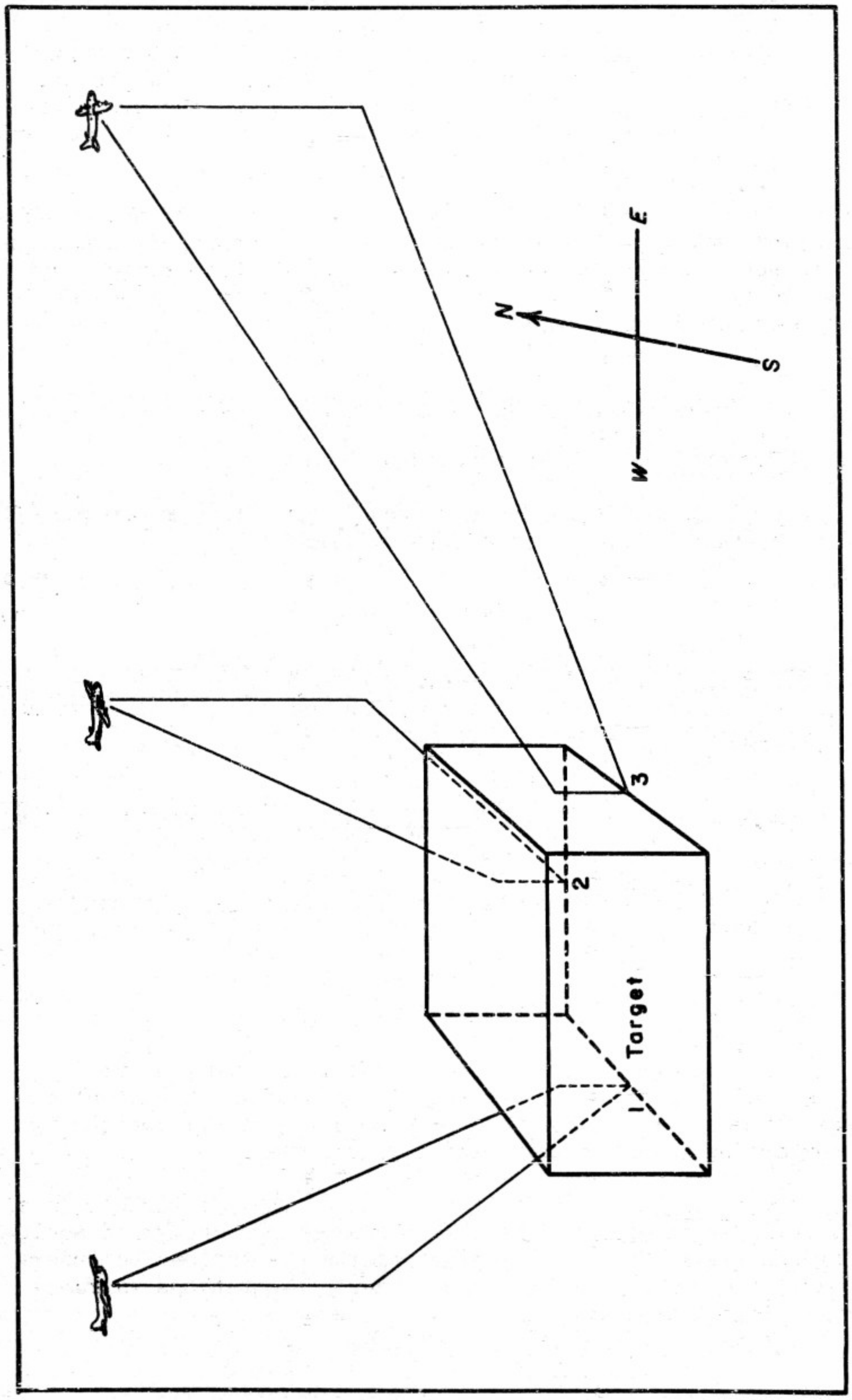


Fig. 21. Aspect fluctuation.

of a building, but will vary considerably depending on the ruggedness of the relief, but the same type of fluctuation will occur. Corresponding to the amount of this fluctuation and the presentations employed to make the identification, the composite target will be plotted out of position.

Such problems as the extremely small scale of the original presentation, the lack of target resolution and aspect fluctuation of the target prohibit further investigation of contouring except on a theoretical basis. Emphasis might better be placed on the development of planimetric mapping.

IV. CONCLUSIONS AND RECOMMENDATIONS

7. Conclusions. It is concluded that:

a. In most cases encountered in practice, radar relief displacements may be approximated by the formula.

$$d_R = h \sin \Phi = \frac{hH}{\sqrt{H^2 + R^2}} = \frac{hH}{S} \qquad (5)$$

b. In those cases where a more accurate solution is desired the second approximation becomes:

$$d_R = h \sin \Phi' = \frac{h(H - h/2)}{\sqrt{(H - h/2)^2 + R^2}} = \frac{h(H - h/2)}{S_M} \qquad (4)$$

c. A usable approximation for relief computations from radar parallax is given by a single solution of the iteration formula:

$$h_{n+1} = \frac{pS_1S_2}{b(H - h_n/2)} \qquad (32)$$

d. The quality of the data obtained from present day radar presentations does not warrant a quantitative investigation of these formulas, nor the design of an instrument for contouring from overlapping radar presentations.

8. Recommendations. It is recommended that no further development of formulas for radar relief displacement nor design of equipment for contouring from overlapping radar presentations be undertaken until the quality of data from radar presentations warrants additional investigation.

APPENDICES

CONFIDENTIAL SECURITY INFORMATION

APPENDIX A

AUTHORITY

SECURITY CLASSIFICATION CONFIDENTIAL

RESEARCH AND DEVELOPMENT PROJECT CARD (NEW PROJECTS)	2. SEC. C	3. PROJ. NO. 8-97-90-001
1. PROJECT TITLE UTILIZATION OF RADAR PRESENTATIONS FOR TOPOGRAPHIC MAPPING		4. 5. REPORT DATE 12 July 49
6. BASIC FIELD OR SUBJECT Mapping, Charting, and Geodesy	7. SUB FIELD OR SUBJECT SUB GROUP Electronic Surveying Equipment	
8. COGNIZANT AGENCY Office, Chief of Engineers	12. CONTRACTOR AND/OR LABORATORY Engr. Res. & Dev. Laboratories	CONTRACT/W. O. NO.
9. DIRECTING AGENCY Engr. Intell. Div., MO, OCE		
10. REQUESTING AGENCY Office, Chief of Engineers	13. RELATED PROJECTS	17. EST. COMPL. DATES RES. Aug. 1951 DEV. TEST OP. EVAL.
11. PARTICIPATION AND/OR COORDINATION Department of Air Force (C)	14. DATE APPROVED 5 August 1949 by GS, USA	
	15. PRIORITY 2-C | 16. IC-12	FY | 18. FISCAL EST'S. 50 | 3 51 | 15 T | 25
19. Item No. 1424, CETC Meeting #201		

20. REQUIREMENT AND/OR JUSTIFICATION A need exists for research and study on techniques for compiling topographic maps from information obtained with radar and other types of airborne electronic presentations. The development of such procedures may permit mapping to be accomplished from data gathered at night or during overcast conditions. This project may provide methods which will substantially improve combat effectiveness.

21. BRIEF OF PROJECT AND OBJECTIVE

a. REFERENCES:
   (1) Engineering Division, Air Materiel Command, Memorandum Report, "Radar Mapping Project of Peru-Ecuador Boundary", dated 30 April 1946.
   (2) War Department Equipment Board Report, dated 29 May 1946, Section XIX, Paragraph 1, General.
   (3) Training Manual, "Interpretation of Radar P.P.I. Scope Photographs", Office of the Assistant Chief of Air Staff-2, Hq, USAAF, dated June 1946.
   (4) Report entitled "Mapping by Radar" by Lt. Howard P. Smith, Jr. U.S.A.F., Randolph Field, Texas, dated 2 February 1948.
   (5) Report entitled "Topofax" by Haller, Raymond, and Brown, Inc., State College, Pa., dated 8 Sept 1948, for Air Materiel Command, Wright Field.

b. OBJECTIVE:
   (1) To obtain knowledge and keep abreast of the art of obtaining information from radar and other types of airborne electronic presentations in order to determine their possible application and utilization for topographic mapping.

c. MILITARY CHARACTERISTICS:
   (1) Not applicable.

d. DISCUSSION:
   (1) The constant development of airborne electronic equipment and flying procedures requires parallel mapping research if maximum utilization of the equipment is to be realized (reference 21a(2)). The ability to utilize radar equipment successfully in gathering the information needed for mapping would permit the scheduling of missions for any time of the day or night, and would eliminate many mapping delays now attributed to poor photographic weather. Research for determining mapping requirements and for developing mapping procedures and techniques is needed.

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SECURITY CLASSIFICATION CONFIDENTIAL

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not only to prepare for possible wartime mapping, but also to accomplish as expeditiously as possible military peacetime mapping.

(2) References 21a(1), 21a(3), and 21a(4) indicate that small scale aeronautical charts can now be made from radar photography. Improvements in equipment and procedures may result in such corresponding increases in the attainable accuracy and detail interpretation that the preparation of topographic maps will prove feasible.

(3) Reference 21a(5) shows that proposals are now being made for increasing still further the topographic information obtainable from airborne electronic devices.

(4) The successful prosecution of all contemplated phases of research and development on this project, will require continuous coordination with the Air Force. Necessary liaison will be maintained through personnel and facilities already provided by both services for this purpose.

(5) Agencies interested in this project, in addition to the Corps of Engineers, are the Army Field Forces and the Department of the Air Force

e. PROJECT PLAN:

(1) It is proposed to carry out research on both standard and new photogrammetric techniques to determine the best procedures for mapping with information obtained from radar and other types of airborne electronic devices. A study of the perspective properties of the presentations (photographs of PPI scopes, etc.), the correlation between scale and the identification of map detail, and the operational principles involved in obtaining this new type of information will be included in the investigation.

(2) As improvements in radar equipment and operating procedures indicate greater map accuracy or modified mapping techniques, additional airborne tests will be requested. Investigations will be made to determine what modifications of methods and equipment or what new items of equipment are necessary or desirable to overcome any deficiencies brought out by these tests. When modifications involve the improvement of radar equipment, appropriate recommendations will be made to the Air Force.

(3) Research and engineering studies required to determine the desired military characteristics of new items of equipment will be made and a separate project or projects to develop the desired equipment will be initiated.

APPENDIX B

JUSTIFICATION OF THE APPROXIMATION IN EQUATION (2)

Equation (2) is given as:

$$H^2 + R^2 = \left(\frac{4d^2Hh - 2d^2h^2 + d^4}{4d^2}\right) + \left(\frac{4h^2H^2 - 4h^3H + h^4}{4d^2}\right) \quad (2)$$

or

$$S^2 = A + B \quad (33)$$

where

$$A = Hh - \frac{h^2}{2} + \frac{d^2}{4} \quad (34)$$

It is desired to substitute d = h in the third term of A. This substitution is made to simplify the results, and is justified on the basis that there is little effect on the final result since the relative numerical values of the quantities involved are generally small.

The range of d is from zero to h. This is illustrated graphically for the extreme cases in Figure 22; d is zero when L is on the perpendicular bisector of BT, and d is equal to h when L lies on the extension of the line BT (or vertically over the target).

As originally derived, A is equal to the expression shown in (34). With the substitution of h^2 for d^2, A becomes

$$A \cong Hh - \frac{h^2}{4} \quad (35)$$

The error in A by this substitution is $h^2/4 - d^2/4$. The maximum value of this error is $h^2/4$ (which occurs when d = 0).

Differentiating equation (33), realizing that B is constant, the expression for the error in this equation is given by

$$2S\Delta S = \Delta A \quad (36)$$

From the previous discussion the maximum error in A is given by $h^2/4$. Thus the percentage of error in S is given by

$$\frac{\Delta S}{S} = \frac{\Delta A}{2S^2} = \frac{h^2}{8S^2} = \frac{1}{2}\frac{(h)^2}{(2S)^2} \quad (37)$$

The relief is generally much smaller than the slant range, and thus the maximum error introduced by substituting d for h in a single term is small. Even in the extreme case where the relief is one-half the slant range, the maximum error in slant range produced by this substitution is only 3 percent. In the general case it will be considerably lower than this as Table I illustrates.

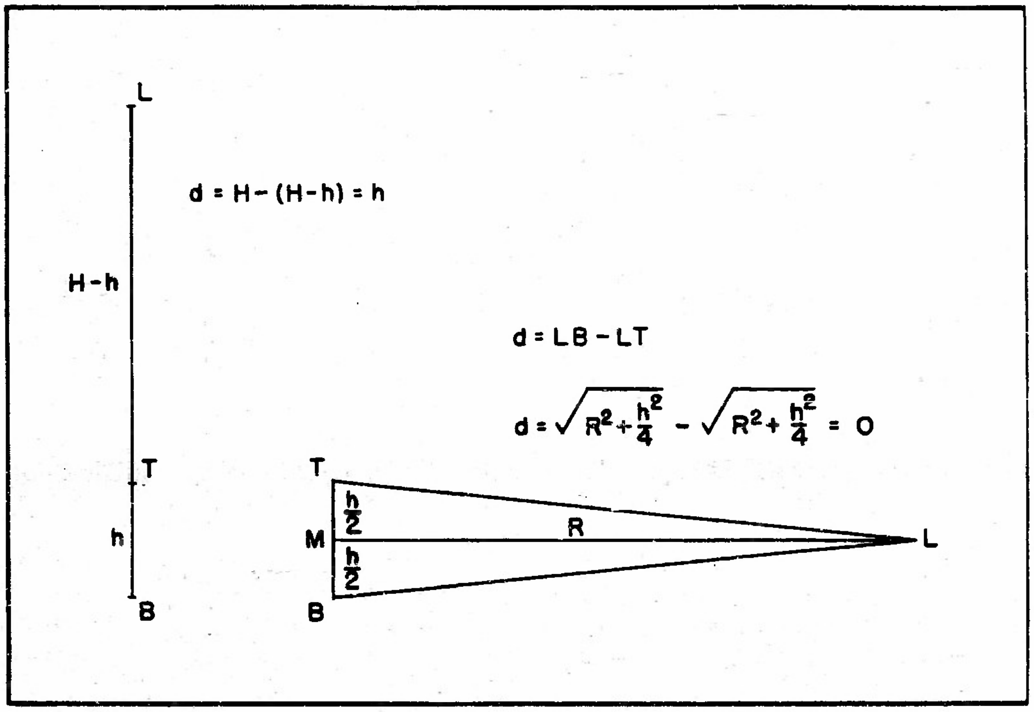

Fig. 22. Range of radar relief displacement.